# Computer Science II - Final exam
# Monday 06 May 2013, 10:00-13:00

This is a closed book exam. Hand calculators are allowed. Show all your work.
Correct answers with insufficient or incorrect work will not get any credit.
Maximum possible score is 100.
**Please solve any FIVE out of the first six questions.**
**Questions 7, 8, 9 are compulsory.**

**Signature:** _____

**Name:** _____

1. (10 points) Let $n > 0$ be an integer and $\alpha > 0$. Show that the Newton's method for solving $x^n - \alpha = 0$ converges for all $x_0$.

2. (10 points) *i*) Exactly how many multiplications are needed to calculate FFT with four data points $(f_0, f_1, f_2, f_3)$. *ii*) Find the Fast Fourier Transform $c = (c_0, c_1, c_2, c_3)$ of the data $f = (2, 0, 2, 0)$ and find the FFT of $c$.

3. (10 points) Recall the definitions $E_k(x) = \exp(ikx)$ and $\langle f, g\rangle_n = (1/n)\sum_{k=0}^{n-1} f(x_k)\overline{g(x_k)}$ where $x_k = 2\pi k/n$. If $f(x_k) = \langle g, E_k\rangle_n$ then prove that $g(x_k) = n\,\langle f, E_k\rangle_n$.

4. (10 points) Determine $a, b, c$ so that the function

$$f(x) = \begin{cases} 3 + x - 9x^2 & \text{if } x \in [0,1] \\ a + b(x-1) + c(x-1)^2 + d(x-1)^3 & \text{if } x \in [1,2] \end{cases}$$

   is a cubic spline with knots $0, 1, 2$. Determine $d$ so that $\int_0^2 [f''(x)]^2\,dx$ is minimized.

5. (10 points) Show that the polynomial interpolation of the data $(x_k, y_k)$ for $k = 0, 1, \ldots, n$ is given by

$$p(x) = \left[\sum_{i=0}^{n} \frac{y_i w_i}{x - x_i}\right]\left[\sum_{i=0}^{n} \frac{w_i}{x - x_i}\right]^{-1} \text{ for } x \neq x_k \text{ where } w_j = \prod_{\substack{i=0 \\ i\neq j}}^{n} (x_i - x_j)^{-1}\ .$$

6. (10 points) Find Lagrange polynomial interpolation of the following data.

| x | 3 | 7 | 1 | 2 |
|---|---|---|---|---|
| y | 10 | 146 | 2 | 1 |

Please turn over.

7. (15 points) Show that the following method for solving the ODE $y' = f(x, y)$ is of $3^{rd}$ order.

$$\begin{aligned} F_1 &= hf(x_k, \eta_k)\,, \\ F_2 &= f(x_k + \frac{1}{2}h, \eta_k + \frac{1}{2}F_1)\,, \\ F_3 &= f(x_k + \frac{3}{4}h, \eta_k + \frac{3}{4}F_2)\,, \\ \eta_{k+1} &= \eta_k + \frac{1}{9}(2F_1 + 3F_2 + 4F_3)\,, \\ x_{k+1} &= x_k + h\,. \end{aligned}$$

8. (10 points) Write an octave function to find the solution of the ODE $y' = f(x, y)$ using backward Euler method. The function input should be the initial condition $x0, y0$, the step-size $h$, the number of steps $N$ to be taken, and the name of the octave function which calculates the right hand side. Use a built-in octave function to calculate roots. The output should be two vectors of length $N$ containing the approximate solution.

9. (6+6+6+7 points) Write octave/matlab commands for the following.

   (a) Create a row vector of length '$n$' of elements equispaced between 1 and 100 (assume '$n$' is a positive integer).

   (b) Define a column vector '$sm$' whose elements are the row sum of a matrix '$A$'.

   (c) Create a vector '$x$' with elements $a_n = [(-1)^n \sin(\pi/n)]/(2n)!$ for $1 \leq n \leq 10$ without using a loop.

   (d) Given the following matlab variables

   ```
   a = [1; 2; 3]; b = [4; 5; 6]; c = [7; 8; 9]; A = [a, b, c]; B = [a, b];
   ```

   which of the following will produce an error message? Why?

   i. `a + b`
   ii. `a*b`
   iii. `a.*b`
   iv. `a.*A`
   v. `B*B`
   vi. `B*A`
   vii. `A*B`